SHORTEST PATH LP

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PROBLEM

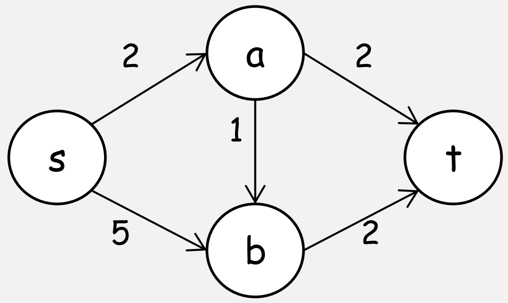
Objective: Complete a Linear Programming (LP) formulation to find the shortest path from a start node (s) to a target node (t) in a given directed graph.

LP Formulation to Complete:

- **Objective Function:** Maximize $d_t - d_s$
- **Constraints:** For every edge (u, v), the condition $d_v \leq d_u + w(u, v)$ must hold.

Tasks:

- Write out the complete LP constraints.
- Set $d_s = 0$.
- Find the optimal maximum value for d_t.
- Identify which path this optimal value corresponds to.

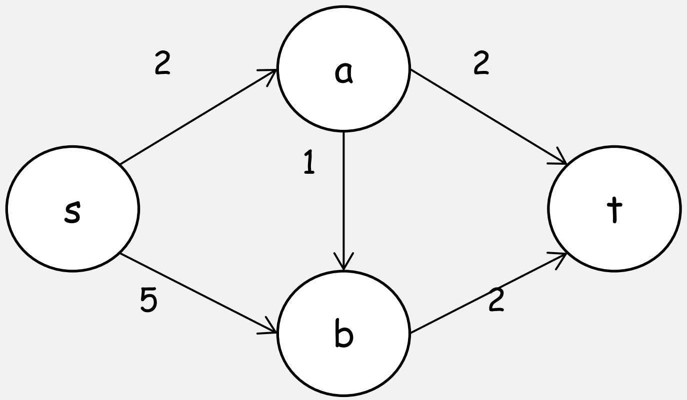


LINEAR PROGRAMMING FORMULATION

Objective Function: $\max\ d_t - d_s$

Subject to: $(d_v \leq d_u + w(u,v))$

- $-d_s + d_a \leq 2$
- $-d_s + d_b \leq 5$
- $-d_a + d_b \leq 1$
- $-d_a + d_t \leq 2$
- $-d_b + d_t \leq 2$
- **Variables:** $d_s, d_a, d_b, d_t \in \mathbb{R}$

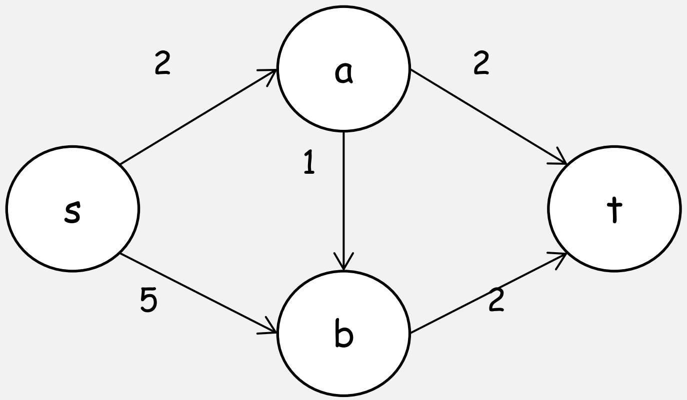


ASSUME $d_s = 0$

Objective Function: $\max \ d_t$

Subject to: $(d_v \leq d_u + w(u, v))$

- $d_a \leq 2$
- $d_b \leq 5$
- $-d_a + d_b \leq 1$
- $-d_a + d_t \leq 2$
- $-d_b + d_t \leq 2$
- **Variables:** $d_s, d_a, d_b, d_t \in \mathbb{R}$



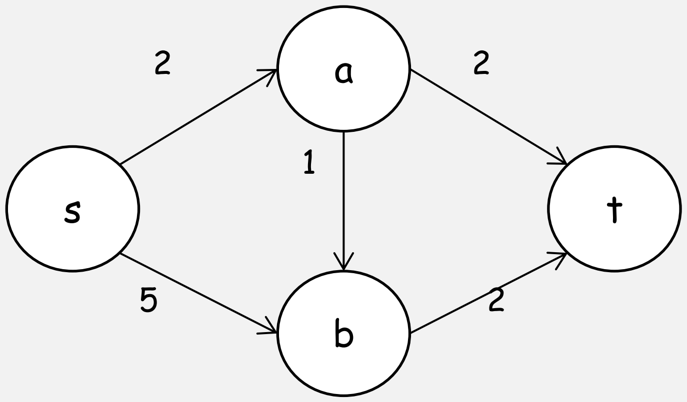
MAXIMUM VALUE FOR d_t

Objective Function: $\max \ d_t$

Subject to: $(d_v \leq d_u + w(u,v))$

- $d_a \leq 2$
- $d_b \leq 5$
- $-d_a + d_b \leq 1$
- $-d_a + d_t \leq 2$
- $-d_b + d_t \leq 2$
- **Variables:** $d_s, d_a, d_b, d_t \in \mathbb{R}$

$\Longrightarrow d_t \leq 4$

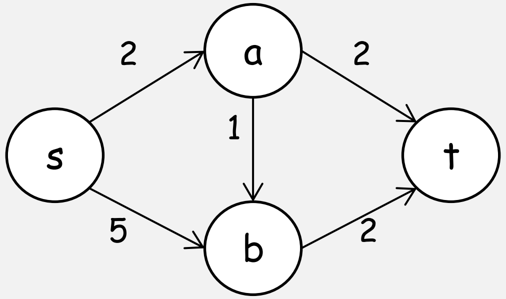


MAXIMUM VALUE FOR d_t

Objective Function: $\max \ d_t$

Subject to: $(d_v \leq d_u + w(u,v))$

- $d_a \leq 2$ (2)
- $d_b \leq 5$
- $-d_a + d_b \leq 1 \Rightarrow d_b \leq 3$ (2)
- $-d_a + d_t \leq 2 \Rightarrow 2 \leq d_a$ (1)
- $-d_b + d_t \leq 2 \Rightarrow 2 \leq d_b$ (1)
- **Variables:** $d_s, d_a, d_b, d_t \in \mathbb{R}$



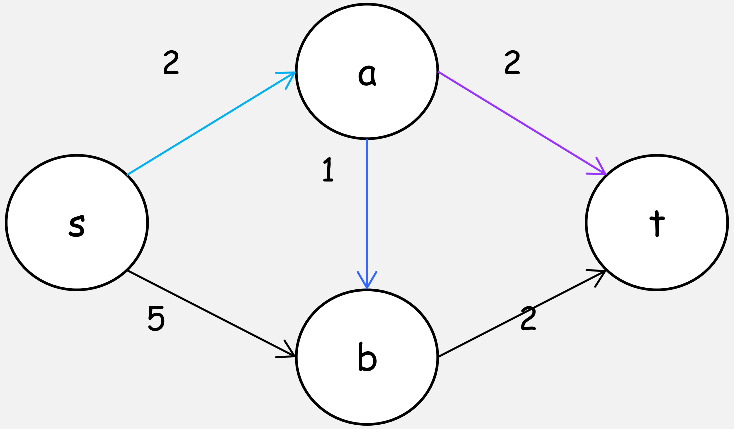
$d_s = 0$
$d_a = 2$
$d_b = 3$
$d_t = 4$

PATH THIS OPTIMAL VALUE

Objective Function: $\max \ d_t$

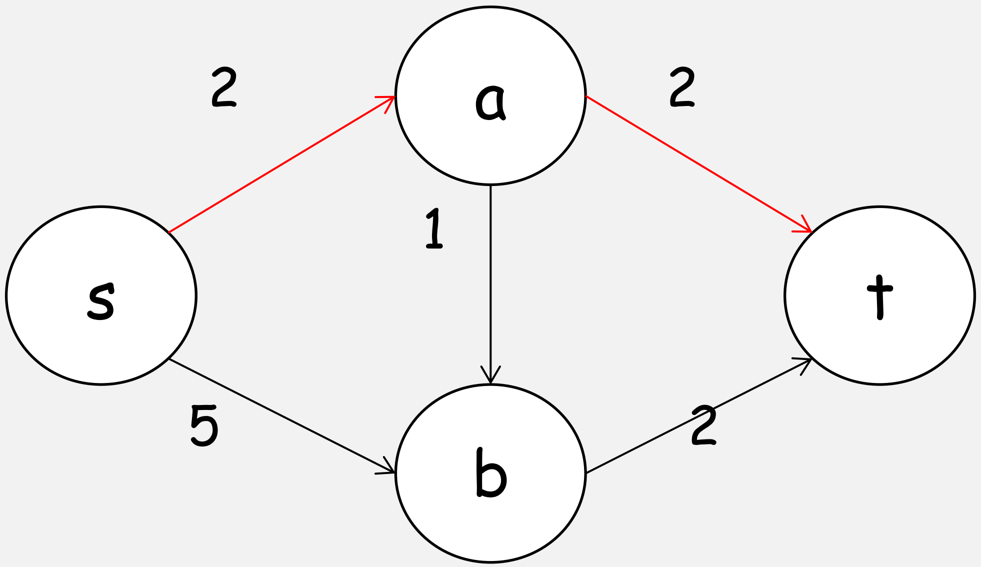
Subject to: $(d_v \leq d_u + w(u,v))$

- $d_a \leq 2$
- $d_b \leq 5$
- $-d_a + d_b \leq 1$
- $-d_a + d_t \leq 2$
- $-d_b + d_t \leq 2$
- **Variables:** $d_s, d_a, d_b, d_t \in \mathbb{R}$



$$
\begin{aligned}
d_s &= 0 \\
d_a &= 2 \\
d_b &= 3 \\
d_t &= 4
\end{aligned}
$$

PATH THIS OPTIMAL VALUE



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